

## Mathematical Model of a Photovoltaic Powered Reverse Osmosis Plant without Batteries

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### Abstract

A small-scale photovoltaic powered reverse osmosis plant is designed to operate at variable flow/pressure in equatorial areas, enabling it to make efficient use of the naturally varying solar resource. The mathematical model for the present plant was developed and experimentally validated, allowing a prediction of the instantaneous operational conditions. The main components of the plant under test are 1 diaphragm DC motor-pump, 1 buck converter (maximum power point tracking), 3 PV modules (55 Wp each) and 1 valve for simulation of the RO membrane.

**Keywords:** Photovoltaic; Simulation, Reverse Osmosis; Brackish Water Desalination.

### 1. Introduction

The water availability all over the world has dropped in such a way that scientific community and international organizations have had special attention to that. According to UNESCO (2003), until half this century water shortage will reach from 2 until 7 billion of people from more than 40 countries. In Brazil, the semi-arid region, which occurs in the Northeast region, has irregular rainfall distribution. Furthermore, approximately 75% of the water from wells has solids concentration above 500 ppm, that is, that water is inadequate for human consumption (MME, 2003).

Desalination of seawater and brackish water is one of the alternatives for ensuring a dependable supply of drinking water. In recent years the process of reverse osmosis (RO) has become a significant technical option to solve this problem through the desalination of brackish and seawater. Some of the reasons for this trend are the low specific energy consumption of this process and the considerable progress made in membrane technology.

The RO process consists, basically, of raw water pumping through selective membranes that retain most of the salts and micro-organisms. Also, photovoltaic (PV) technology is a reliable option for regions without electric power grid, as most of the rural communities in the Northeast of Brazil, where one finds yearly average temperature of 27 °C and a solar potential of about 2000 kWh/m<sup>2</sup>. However, the worldwide number of installations that combine these two technologies (PV + RO) is still very small.

Experiments show that a treatment of water through this combination is technically feasible and nowadays for stand-alone and small scale a photovoltaic powered reverse osmosis (PV-RO) plant is also viable economically.

In the present project a PV-RO plant without batteries is proposed. The nonexistence of batteries decreases the maintenance level, a fundamental aspect in projects for remote areas. However, due to the intermittent PV power supply, it is expected variable pressure/flow conditions.

A mathematical model for the plant was developed and experimentally validated, allowing a prediction of the instantaneous operational conditions. This is particularly important in the study of critical design parameters. In this way, the aim of the project is to achieve an appropriate technology for small-scale desalination plants for remote communities.

Colangelo et al.(1999) have described a mathematical model so that the governing equations can be solved using average hourly solar-insolation and ambient temperature values. Thompson and Infield (2002) have presented an extensive modelling with performance and cost estimates. They have developed a maximum power point tracking for the photovoltaic array, the same was made in this study.

## 2. PV-RO Plant without Batteries: Design and Operation

The process of osmosis, found in organic cells, can be reversed and is then called Reverse Osmosis (RO). This process is used chiefly for separating the solvent (water) from aqueous salt solutions.

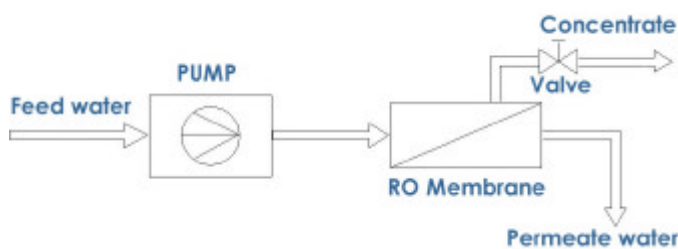


Fig. 1. RO Hydraulic Structure

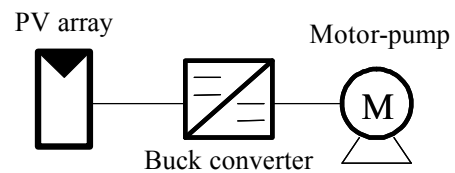


Fig. 2. PV-RO Electrical Structure

The basic RO hydraulic structure is shown in Fig. 1. The process of RO occurs when a feed pressure is greater than its osmotic pressure. The feed water flows through the membrane, leaving 85 to 99 % of the salt behind. Consequently part of the feed water becomes more concentrated brine.

The motor-pump is powered by PV array as illustrated in Fig. 2. An appropriate PV-RO design method was proposed on a previous paper, accordingly to this topology without energy storage (Carvalho et al., 2004a). Because of the intermittent PV power supply, it is expected variable pressure/flow conditions. About these conditions the membrane manufacturers have been reluctant to give warranties regarding their products. However, some information is given to orientate the operation (DOW, 2004):



Fig. 3. PV-RO plant without batteries

- A high start/stop frequency can lower the performance and the lifetime of the membranes;
- During low feed flow operation, a minimum concentrate flow must be maintained.

Permeate water flow and salt rejection are the key performance parameters of a reverse osmosis plant. These parameters are mainly influenced positively by the temperature and pressure, and negatively by the feed water salt concentration.

*A priori* the feed water salt concentration and temperature are uncontrollable and vary along the seasons. Consequently, only two variables (pressure and recovery ratio) can be controllable. Based on that, two operating strategies are proposed:

- operate at constant feed pressure;
- operate at constant recovery ratio.

The main components of the PV-RO desalination plant currently being developed, shown in Fig. 3, are: 1 diaphragm DC motor-pump, 1 buck converter, 3 PV modules (55 Wp each) and 1 valve for simulation of the RO membrane. The simulated RO membrane has a nominal permeate water flow of 50 L/h.

The description of the control method and the maximum power point tracking algorithm developed were presented on a previous paper (Carvalho et al., 2004b). In this paper was shown also the PV cell and buck converter model validation.

### 3. Mathematical Model

For the present research, a model was implemented in Matlab<sup>®</sup> Simulink<sup>®</sup>, as shown in Fig. 4, where the reference data are the global irradiation, the ambient temperature and the feed pressure. The main blocks are: PV panel, DC-DC buck converter (maximum power point tracking), motor-pump DC and RO membrane.

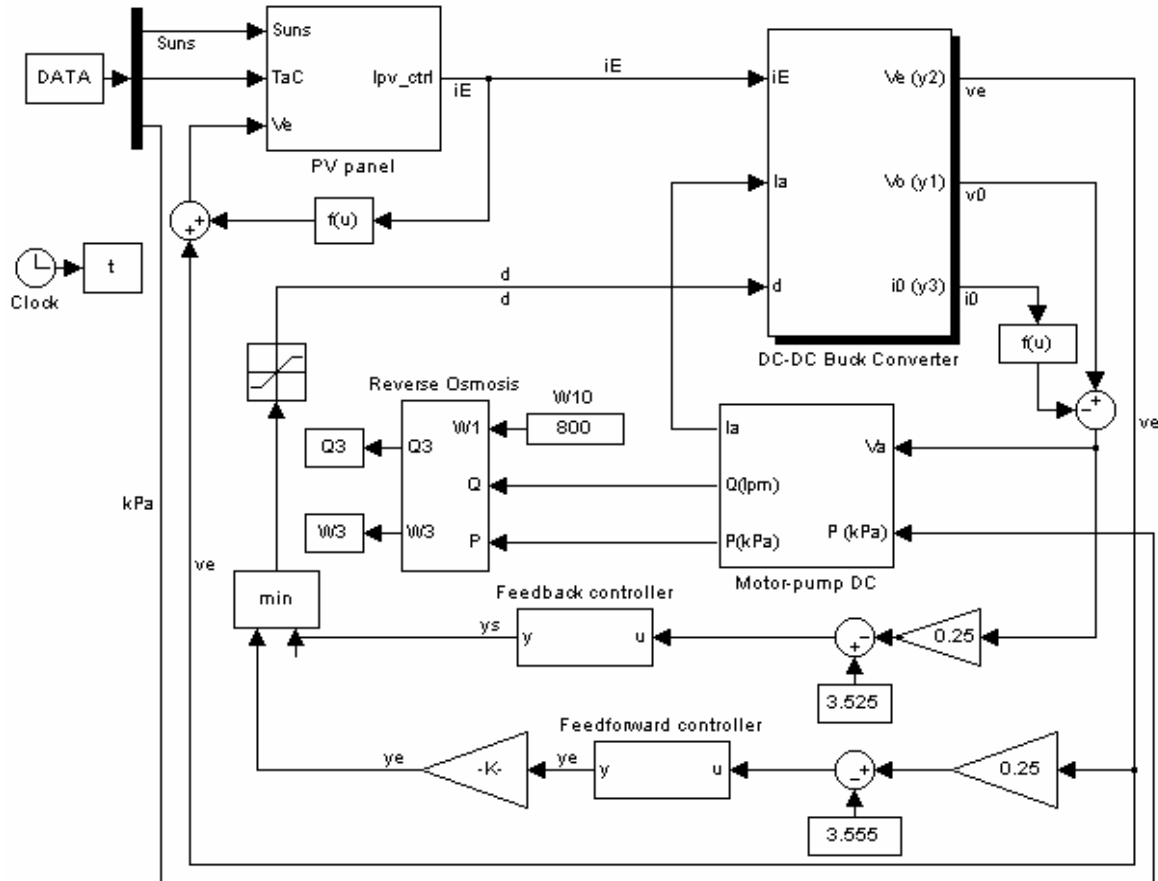


Fig. 4. PV-RO Simulink simulation

### 3.1 Motor-pump

The behavior of the permanent magnet DC motor is described by:

$$V_0 = Km.w + Ra.Ia \quad (1)$$

$$T_e = Km.Ia \quad (2)$$

$$\frac{dw}{dt} = \frac{T_e - T_f - T_{LOAD}}{J} \quad (3)$$

where and  $V_0$  is the motor input voltage [V];  $Ra$ ,  $Ia$  are the armature resistance [O] and current [A];  $w$  is the angular speed [rad/s];  $Km$  is a motor constant [N.m/A];  $T_e$ ,  $T_f$ ,  $T_{LOAD}$  are the electrical, no load and load torque [N.m] and  $J$  is the total inertia [kg.m<sup>2</sup>].

To archive the pump characteristics, some experimental data are analyzed and it is observed that the diaphragm positive displacement pump used have a direct relationship between the armature current and the pressure  $P$  [kPa]:

$$P = 82.Ia - 71 \quad (4)$$

It simplifies the simulation, because  $T_{LOAD}$  is calculated to represent the reference pressure (input data).

### 3.2 REVERSE OSMOSIS

Henne (1980) worked with a RO mathematical description, called diffuse solution model, to describe the material transportation through the RO membrane. From this model can be found the specific water flow  $V_w$  [(m<sup>3</sup>/s)/m<sup>2</sup>]:

$$V_w = A.(p_1 - \Delta p), \quad (5)$$

where the osmotic pressure  $\Delta p$  is given by:

$$\Delta p = b.(w_m - w_3). \quad (6)$$

The specific salt flow  $V_s$  is:

$$V_s = B.(w_m - w_3) \quad (7)$$

where  $b$  is a osmotic pressure constant,  $w_m$  is the salt concentration on the membrane and  $w_3$  in the permeate,  $p_1$  is the feed pressure. The membrane constants  $A$  and  $B$  have influence from the temperature and the pressure. About the temperature influence, Rautenbach and Albrecht (1981) set up the equations:

$$A = A_0 . e^{\mathbf{a}_T \frac{T-T_0}{T_0}} \quad (8)$$

$$B = B_0 . e^{\mathbf{b}_T \frac{T-T_0}{T_0}} \quad (9)$$

where  $T_0 = 293$  K,  $\mathbf{a}_T = 7,08$  and  $\mathbf{b}_T = 3,0$ , all of that obtained empirically. The permeate flow grows at about 2 % per grad (Böddeker, 1984). The pressure influence in  $A$  and  $B$  is described by:

$$A = A_0 . e^{\mathbf{a}_p \frac{\Delta p}{p_0}} \quad (10)$$

$$B = B_0 . e^{\mathbf{b}_p \frac{\Delta p}{p_0}} \quad (11)$$

where  $p_0 = 100$  kPa,  $\mathbf{a}_p = -0,004$  and  $\mathbf{b}_p \cong 0$ , demonstrating that the pressure influence in the specific salt flow is very small.

With the water flow through the membrane there is a transportation of salts. Consequently, a great salt concentration appears at the membrane-surface. This appearance is called "concentration-polarization" and the material-balance is determined by:

$$\frac{w_m - w_3}{w_1 - w_3} = \exp\left(\frac{V_w}{k}\right) \quad (12)$$

where  $w_1$  is the salt concentration in the feed water. The so-called material transition coefficient  $k$  is empirically found and depends of the current condition along the membrane. From the equations (5) and (6), together with equation (12), an iterative expression for the calculation of the specific water flow under consideration of the concentration polarization is:

$$V_w = A \cdot \left[ p_1 - b \cdot \exp\left(\frac{V_w}{k}\right) \cdot (w_1 - w_3) \right] \quad (13)$$

Since  $V_s \ll V_w$ , than:

$$w_3 = \frac{V_s}{V_s + V_w} \approx \frac{V_s}{V_w} \quad (14)$$

Together with the equations (7) and (12) can be found the permeate salt concentration:

$$w_3 = \frac{B \cdot w_1 \cdot \exp\left(\frac{V_w}{k}\right)}{V_w + B \cdot \exp\left(\frac{V_w}{k}\right)} \quad (15)$$

#### 4. Simulation Results and Experimental Validation

The following simulation results are derived from Matlab-Simulink<sup>®</sup> model of the PV-RO plant, and they are compared with the experimental results. The simulation of the RO membrane, model XLE-2540, are compared with the results of a manufacturer simulation software called ROSA<sup>®</sup>.

The first PV-RO plant operation hours on August 28th 2004 are shown in Fig. 5. The measurements were used as reference data for the simulation model and both measured and calculated (with marks) results are plotted in this figure. The relevance of the model is indicated by the coincidence of the values. Some volumetric pumps characteristics can be noted, as the water flow linearity and the relationship between the power supply, in this case the solar radiation (orange line), and the pressure (purple line). From the feed water of 800 ppm the system produces an average permeate water flow of about 29 L/h with 44 ppm. It is a relatively good response since the maximum of 50 L/h to preserve the maximum recovery ratio (relation between permeate water flow and feed water flow) of 15 % stipulated by the membrane manufacturer. However, in some short periods are noted that the recovery ratio exceeds this value. An experimental probe is needed to describe how it affects the RO membrane.

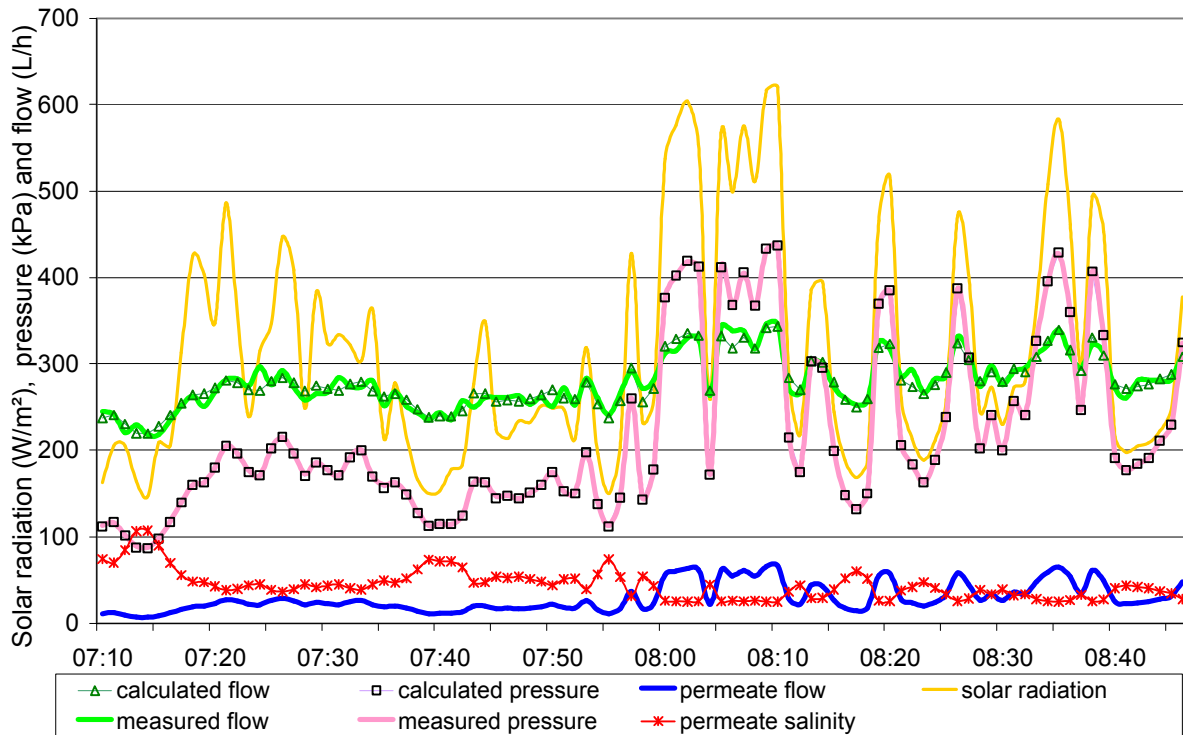


Fig. 5. PV-RO plant without batteries: Measured and calculated (with marks) results

## 5. Conclusion

A small-scale PV-RO plant with variable flow/pressure (without batteries) was simulated. The mathematical model implemented in Matlab-Simulink<sup>®</sup> have three input variables: the global radiation, ambient temperature and the feed pressure. It was used the diffuse solution model to describe the RO membrane. In the DC motor pump equations, one simplification was made: the direct relationship between the armature current and the pressure was used to calculate the load torque.

The laboratory measurements have validated the implemented model, except for the permeate water flow and salinity that was validated by the software called ROSA<sup>®</sup>.

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